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$B \to \eta_c K (\eta_c' K)$ decays in QCD factorization

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Abstract. We study the exclusive decays of the B meson into pseudoscalar charmonium states η_c and η_c' within the QCD factorization approach and find that the non-factorizable corrections to naive factorization are infrared safe at leading-twist order. The spectator interactions arising from the kaon twist-3 effects are formally power suppressed but chirally and logarithmically enhanced. An important improvement by including the $\mathcal{O}(\alpha_s)$ corrections is the cancellation of the renormalization scale μ dependence of the decay amplitude. However, the calculated decay rates are too small to accommodate the experimental data. On the other hand, we compare the theoretical calculations for B meson decays to $J/\psi, \psi', \eta_c$ and η_c', and find that the predicted relative decay rates of these four states are approximately compatible with the experimental data.

1 Introduction

Exclusive decays of the B meson to charmonium are important since those decays, e.g. $B \to J/\psi K$, are regarded as the golden channels for the study of CP-violation in B decays. However, quantitative understanding of these decays is difficult due to the strong-interaction effects. It is argued that because the size of the charmonium is small ($\sim 1/\alpha_s m_\psi$) and its overlap with the (B, K) system is negligible [1], the same QCD-improved factorization method as for $B \to \pi\pi$ [2] can be used for $B \to J/\psi K$ decay.

The general idea of QCD factorization is that in the heavy quark limit $m_b \gg \Lambda_{\rm QCD}$, the transition matrix elements of operators in the hadronic decay $B \to M_1 M_2$ with M_1 being the recoiled meson and M_2 being the emitted meson are given by [2]

$$\begin{aligned}
&\langle M_1 M_2 | O_i | B \rangle \\
&= \langle M_1 | j_1 | B \rangle \langle M_2 | j_2 | 0 \rangle \left[1 + \sum r_n \alpha_s^n + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b} \right) \right] \\
&= \sum_j F_j^{BM_1}(m_2^2) \int_0^1 {\rm d}u \, T_{ij}^{\rm I}(u) \phi_{M_2}(u) \qquad (1) \\
&\quad + \int_0^1 {\rm d}\xi \, {\rm d}u \, {\rm d}v \, T^{\rm II}(\xi, u, v) \phi_B(\xi) \phi_{M_1}(v) \phi_{M_2}(u),
\end{aligned}$$

where M_2 is a light meson or a quarkonium and F^{BM_1} is the $B \to M_1$ transition form factor. Here ϕ_M is the meson light-cone distribution amplitude and $T^{\rm I,II}$ are perturbatively calculable hard scattering kernels. If we neglect strong-interaction corrections, (1) reproduces the result of naive factorization in which the decay amplitude is proportional to F^{BM_1} multiplied by the M_2 decay constant. However, hard gluon exchange between the M_2 and the BM_1 system implies a non-trivial convolution of hard scattering kernels $T^{\rm I,II}$ with the distribution amplitude ϕ_{M_2}.

Indeed, the explicit calculations [3,4] for $B \to J/\psi K$ showed that the non-factorizable vertex contribution is infrared safe and the spectator contribution is perturbatively calculable at twist-2 order. The small size argument for the applicability of QCD factorization for charmonia is intuitive, and it needs verifying for charmonium states other than the J/ψ and $\psi'(\psi(2S))$. For instance, it will be useful to examine how QCD factorization works for B exclusive decays to the pseudoscalar charmonium states η_c and $\eta_c'(\eta_c(2S))$, e.g. about the infrared safety of the vertex correction, the renormalization scale dependence of the decay amplitude, as well as the numerical predictions for the decay rates (in contrast, there exist infrared divergences in B exclusive decays into P-wave charmonium states [5]).

Experimentally, for the pseudoscalar charmonium state η_c, the $B \to \eta_c K$ decay has been observed by CLEO [6], BaBar [7], and Belle [8] with relatively large branching fractions. Moreover, very recently the $\eta_c'(\eta_c(2S))$ meson has also been observed in the $B \to \eta_c' K$ decay by Belle [9]. So it is interesting to compare the predictions of these decay modes into pseudoscalar charmonium based on the QCD factorization approach with the experimental data.

Our paper is organized as follows. In Sect. 2, we give the results for vertex and spectator corrections to $B \to \eta_c K$ within the QCD factorization approach. Contributions from the kaon twist-3 light-cone distribution amplitude and numerical results are also included. In Sect. 3, we give the results for $B \to \eta_c' K$ decay. In the last section, we compare the relative theoretical decay rates for $J/\psi, \psi', \eta_c$ and η_c' with the experimental data. Then we will have some discussion to finish our paper.

2 $B \to \eta_c K$ decay within QCD factorization

We now consider $\overline{B} \to \eta_c K$ decay. The effective Hamiltonian for this decay mode is written as [10]

$$H_{\text{eff}} = \frac{G_{\text{F}}}{\sqrt{2}} \left(V_{cb} V_{cs}^* (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2) - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i \mathcal{O}_i \right), \tag{2}$$

where C_i are the Wilson coefficients and $V_{q_1 q_2}$ are the CKM matrix elements. The relevant operators $\mathcal{O}_i$ in H_{eff} are given by

$$\begin{aligned} \mathcal{O}_1 &= (\bar{s}_\alpha b_\beta)_{V-A} \cdot (\bar{c}_\beta c_\alpha)_{V-A}, \\ \mathcal{O}_2 &= (\bar{s}_\alpha b_\alpha)_{V-A} \cdot (\bar{c}_\beta c_\beta)_{V-A}, \\ \mathcal{O}_{3(5)} &= (\bar{s}_\alpha b_\alpha)_{V-A} \cdot \sum_q (\bar{q}_\beta q_\beta)_{V-A(V+A)}, \\ \mathcal{O}_{4(6)} &= (\bar{s}_\alpha b_\beta)_{V-A} \cdot \sum_q (\bar{q}_\beta q_\alpha)_{V-A(V+A)}, \\ \mathcal{O}_{7(9)} &= \frac{3}{2} (\bar{s}_\alpha b_\alpha)_{V-A} \cdot \sum_q e_q (\bar{q}_\beta q_\beta)_{V+A(V-A)}, \\ \mathcal{O}_{8(10)} &= \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \cdot \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A(V-A)}, \end{aligned} \tag{3}$$

where $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$ and the sum over q runs over u, d, s, c and b.

To calculate the decay amplitude, we introduce the η_c decay constant as

$$\langle \eta_c(p) | \bar{c}(0) \gamma_\mu \gamma_5 c(0) | 0 \rangle = -\mathrm{i} f_{\eta_c} p_\mu, \tag{4}$$

where f_{η_c} is the η_c decay constant which can be estimated from the QCD sum rules or potential models.

The leading-twist light-cone distribution amplitude of η_c is then expressed compactly as

$$\begin{aligned} &\langle \eta_c(p) | \bar{c}_\alpha(z_2) c_\beta(z_1) | 0 \rangle \\ &= \frac{\mathrm{i} f_{\eta_c}}{4} \int_0^1 \mathrm{d}u \cdot \mathrm{e}^{\mathrm{i}(u p \cdot z_2 + (1-u) p \cdot z_1)} \left[(\not{p} - m_{\eta_c}) \gamma_5 \right]_{\beta\alpha} \phi_{\eta_c}(u), \end{aligned} \tag{5}$$

where u and $1-u$ are respectively the momentum fractions of the c and $\bar{c}$ quarks inside the η_c meson, and the wave function $\phi_{\eta_c}(u)$ for η_c meson is symmetric under $u \leftrightarrow 1-u$.

As for the kaon light-cone distribution amplitudes, we will follow [3] in choosing

$$\begin{aligned} \langle K(p) | \bar{s}_\beta(z_2)\, d_\alpha(z_1) | 0 \rangle &= \frac{\mathrm{i} f_K}{4} \int_0^1 \mathrm{d}x\, \mathrm{e}^{\mathrm{i}(x\, p \cdot z_2 + (1-x)\, p \cdot z_1)} \\ &\times \Bigg\{ \not{p} \gamma_5\, \phi_K(x) \\ &- \mu_K \gamma_5 \left[\phi_K^p(x) - \sigma_{\mu\nu}\, p^\mu (z_2 - z_1)^\nu\, \frac{\phi_K^\sigma(x)}{6} \right] \Bigg\}_{\alpha\beta}, \end{aligned} \tag{6}$$

where x and $1-x$ are respectively the momentum fractions of the s and $\bar{d}$ quarks inside the K meson. The asymptotic limit of the leading-twist distribution amplitude is $\phi_K(x) = 6x(1-x)$. We also use the asymptotic forms $\phi_K^p(x) = 1$ and $\phi_K^\sigma(x) = 6x(1-x)$ for the kaon twist-3 two-particle distribution amplitudes. The chirally-enhanced factor is written as $r_\chi^K = 2\mu_K/m_b = 2m_K^2/m_b(m_s + m_d)$, which is formally of order Λ_{QCD}/m_b but numerically close to unity.

In naive factorization, we neglect the strong-interaction corrections and the power corrections in Λ_{QCD}/m_b. Then the decay amplitude is written as

$$\begin{aligned} \mathrm{i} M_0 &= \mathrm{i} f_{\eta_c} m_B^2 F_0(m_{\eta_c}^2) \frac{G_{\text{F}}}{\sqrt{2}} \\ &\times \left[V_{cb} V_{cs}^* \left(C_2 + \frac{C_1}{N_c} \right) - V_{tb} V_{ts}^* \left(C_3 + \frac{C_4}{N_c} - C_5 - \frac{C_6}{N_c} \right) \right], \end{aligned} \tag{7}$$

where N_c is the number of colors. We do not include the effects of the electroweak penguin operators since they are numerically small.

The form factors for $\overline{B} \to K$ are given as

$$\begin{aligned} &\langle K(p_K) | \bar{s} \gamma_\mu b | B(p_B) \rangle \\ &= \left[(p_B + p_K)_\mu - \frac{m_B^2 - m_K^2}{p^2} p_\mu \right] F_1(p^2) \\ &\quad + \frac{m_B^2 - m_K^2}{p^2} p_\mu F_0(p^2), \end{aligned} \tag{8}$$

where $p = p_B - p_K$ is the momentum of η_c with $p^2 = m_{\eta_c}^2$ and we will neglect the kaon mass for simplicity.

As we can see easily in (7), this amplitude is unphysical because the Wilson coefficients depend on the renormalization scale μ while the decay constant and the form factors are independent of μ. This is the well known problem with naive factorization. However, we will show later that the μ dependence of the Wilson coefficients is cancelled and the overall amplitude is insensitive to the renormalization scale if we include the non-factorizable α_{s} corrections.

Taking the non-factorizable order α_{s} strong-interaction corrections in Fig. 1 into account, the full decay amplitude for $\overline{B} \to \eta_c K$ within the QCD factorization approach is written as

$$\mathrm{i} M = \mathrm{i} f_{\eta_c} m_B^2 F_0(m_{\eta_c}^2) \frac{G_{\text{F}}}{\sqrt{2}} \left[V_{cb} V_{cs}^* a_2 - V_{tb} V_{ts}^* (a_3 - a_5) \right], \tag{9}$$

where the coefficients a_i $(i = 2, 3, 5)$ in the naive dimension regularization (NDR) scheme are given by

$$\begin{aligned} a_2 &= C_2 + \frac{C_1}{N_c} + \frac{\alpha_{\text{s}}}{4\pi} \frac{C_F}{N_c} C_1 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_{\text{I}} + f_{\text{II}} \right), \\ a_3 &= C_3 + \frac{C_4}{N_c} + \frac{\alpha_{\text{s}}}{4\pi} \frac{C_F}{N_c} C_4 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_{\text{I}} + f_{\text{II}} \right), \\ a_5 &= C_5 + \frac{C_6}{N_c} - \frac{\alpha_{\text{s}}}{4\pi} \frac{C_F}{N_c} C_6 \left(-6 + 12 \ln \frac{m_b}{\mu} + f_{\text{I}} + f_{\text{II}} \right), \end{aligned} \tag{10}$$

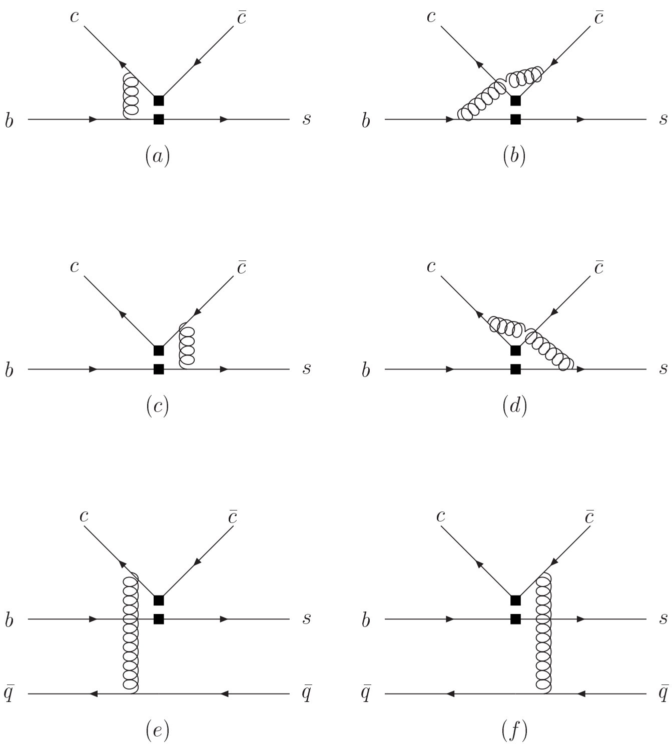

Fig. 1a–f. Feynman diagrams for non-factorizable corrections to $\overline{B} \to \eta_c K$ decay

where $C_F = (N_c^2 - 1)/(2N_c)$.

The function $f_{\rm I}$ in (10) is calculated from the four vertex corrections a, b, c, and d in Fig. 1 and it reads

$$f_{\rm I} = \int_0^1 \mathrm{d}u\ \phi_{\eta_c}(u) \times \left[\frac{3(1-2u)}{1-u} \ln[u] + 3(\ln(1-z) - \mathrm{i}\pi) - \frac{2z(1-u)}{1-zu} - \frac{2uz(\ln[1-z] - \mathrm{i}\pi)}{1-(1-u)z} - \frac{u^2 z^2 (\ln[1-z] - \mathrm{i}\pi)}{(1-(1-u)z)^2} \right.$$

$$\left. + uz^2 \ln[uz] \left(\frac{u}{(1-(1-u)z)^2} - \frac{1-u}{(1-uz)^2} \right) + 2uz \ln[uz] \left(\frac{1}{1-(1-u)z} - \frac{1}{1-uz} \right) \right], \qquad (11)$$

where $z = m_{\eta_c}^2 / m_B^2$, and we have already symmetrized the result with respect to $u \leftrightarrow 1-u$.

The function $f_{\rm II}$ in (10) is calculated from the two spectator interaction diagrams e and f in Fig. 1 and it is given by

$$f_{\rm II} = \frac{4\pi^2}{N_c} \frac{f_K f_B}{m_B^2 F_0(m_{\eta_c}^2)} \int_0^1 \mathrm{d}\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 \mathrm{d}u \frac{\phi_{\eta_c}(u)}{u} \times \int_0^1 \frac{\mathrm{d}x}{x} \left[\phi_K(x) + \frac{2\mu_K \phi_K^p(x)}{m_b(1-z)} \right], \qquad (12)$$

where ϕ_B is the light-cone wave function for the B meson. The spectator contribution depends on the wave function ϕ_B through the integral

$$\int_0^1 \mathrm{d}\xi \frac{\phi_B(\xi)}{\xi} \equiv \frac{m_B}{\lambda_B}. \qquad (13)$$

Since $\phi_B(\xi)$ is appreciable only for ξ of order $\Lambda_{\rm QCD}/m_B$, λ_B is of order $\Lambda_{\rm QCD}$. We will follow [3] to choose $\lambda_B \approx 300\,\mathrm{MeV}$ in the numerical calculations.

There is an integral in (12) arising from kaon twist-3 effects, which will give a logarithmic divergence. Following [3], we treat the divergent integral as an unknown parameter and write

$$\int_0^1 \mathrm{d}x \frac{\phi_K^p(x)}{x} = \int_0^1 \frac{\mathrm{d}x}{x} = X_H, \qquad (14)$$

where $\phi_K^p(x) = 1$ is used for the kaon twist-3 light-cone distribution amplitude. We will choose $X_H = \ln(m_B/\Lambda_{\rm QCD}) \approx 2.4$ as a rough estimate in our calculation.

For numerical analysis, we choose $F_0(m_{\eta_c}^2) = 0.41$ [11] and use the following input parameters:

$$m_b = 4.8\,\mathrm{GeV}, \quad m_B = 5.28\,\mathrm{GeV}, \quad m_{\eta_c} = 3.0\,\mathrm{GeV},$$
$$f_{\eta_c} = 350\,\mathrm{MeV}\ [12], \quad f_B = 180\,\mathrm{MeV}, \quad f_K = 160\,\mathrm{MeV}. \qquad (15)$$

The asymptotic form of the distribution amplitude $\phi_{\eta_c}(u)$ is given as $\phi_{\eta_c}(u) = 6u(1-u)$. In the numerical analysis, we also consider the form $\phi_{\eta_c}(u) = \delta(u - 1/2)$, which comes from the naive expectation of the distribution amplitude. Although there are uncertainties associated with the form of the wave function, we will see shortly that the calculated decay rates are not very sensitive to the choice of the distribution amplitude. The results of the coefficients a_i are listed in Table 2.

Table 1. Leading-order (LO) and next-to-leading-order (NLO) Wilson coefficients in the NDR scheme [10] with $\mu = 4.4\,\mathrm{GeV}$ and $\Lambda_{\overline{\rm MS}}^{(5)} = 225\,\mathrm{MeV}$

	C_1	C_2	C_3	C_4	C_5	C_6
LO	1.144	−0.308	0.014	−0.030	0.009	−0.038
NDR	1.082	−0.185	0.014	−0.035	0.009	−0.041

Table 2. The coefficients a_i at $\mu = 4.4\,\mathrm{GeV}$ with different choices of $\phi_{\eta_c}(u)$

$\phi_{\eta_c}(u)$	a_2	a_3	a_5
$6u(1-u)$	$0.1043 - 0.0684\mathrm{i}$	$0.0045 + 0.0022\mathrm{i}$	$-0.0035 - 0.0026\mathrm{i}$
$\delta(u-1/2)$	$0.0792 - 0.0682\mathrm{i}$	$0.0055 + 0.0022\mathrm{i}$	$-0.0048 - 0.0026\mathrm{i}$

With the help of these coefficients a_i, we calculated the decay branching ratios. For $\phi_{\eta_c}(u) = 6u(1-u)$ we got $\mathrm{Br}(\overline{B} \to \eta_c K) = 1.9 \times 10^{-4}$, and for $\phi_{\eta_c}(u) = \delta(u - 1/2)$ we got $\mathrm{Br}(\overline{B} \to \eta_c K) = 1.4 \times 10^{-4}$.

The measured branching ratios are

CLEO Collaboration [6]:

$$\mathrm{Br}(B^0 \to \eta_c K^0) = (1.09^{+0.55}_{-0.42}) \times 10^{-3},$$

BaBar Collaboration [7]: (16)

$$\mathrm{Br}(B^0 \to \eta_c K^0) = (1.18 \pm 0.16 \pm 0.13 \pm 0.37) \times 10^{-3},$$

Belle Collaboration [8]:

$$\mathrm{Br}(B^0 \to \eta_c K^0) = (1.23 \pm 0.23) \times 10^{-3},$$

which are about 6–9 times larger than our theoretical results.

We show the renormalization scale dependence of the branching ratio in Fig. 2. Results within the QCD factorization approach with different choices of the wave function for η_c and results without non-factorizable corrections are all shown in the figure. We also present central values with different experiments for comparison. As mentioned before, the result of naive factorization is unphysical and very sensitive to the renormalization scale, and this is clearly seen in Fig. 2. But when we include non-factorizable α_s corrections, the dependence of the branching ratio on the renormalization scale μ becomes very mild.

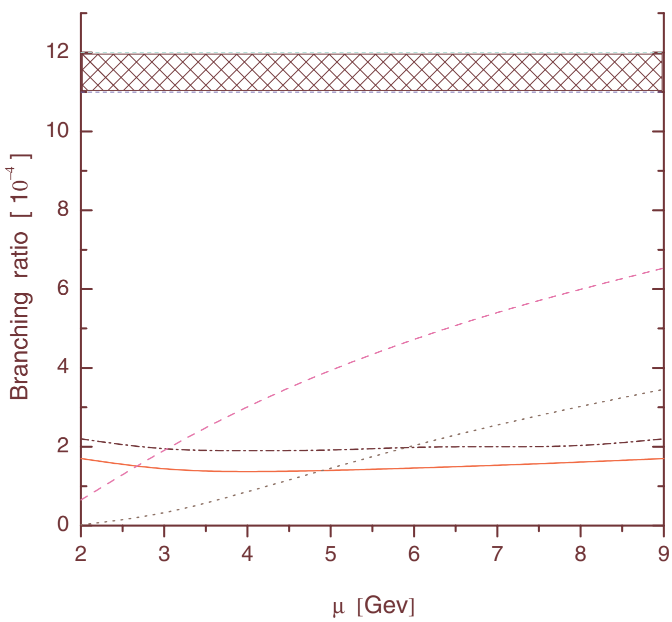

Fig. 2. Dependence of the branching ratio for $\overline{B} \to \eta_c K$ decay on the renormalization scale μ with $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 225\,\mathrm{MeV}$. The solid line and the dash-dotted line represent choices of the wave function $\phi_{\eta_c}(u) = 6u(1-u)$ and $\phi_{\eta_c}(u) = \delta(u-1/2)$, respectively. Results without non-factorizable corrections are shown by the dotted line and the dashed line with choices of LO and NLO Wilson coefficients respectively. The netted band represents central values with different experiments

3 $B \to \eta_c' K$ decay

In the last section, we presented the detailed calculations for $\overline{B} \to \eta_c K$ decay within the QCD factorization approach. The calculation of the branching ratio for $\overline{B} \to \eta_c' K$ decay is similar to that for the η_c given above, and we need not repeat the calculations in this section. Here we give a rough estimate, which is close to a detailed calculation by us, for this decay channel from the decay rate ratio of η_c' to η_c:

$$\begin{aligned} &\frac{\mathrm{Br}(B^0 \to \eta_c' K^0)}{\mathrm{Br}(B^0 \to \eta_c K^0)} \\ &\approx \left(\frac{f_{\eta_c'}}{f_{\eta_c}}\right)^2 \cdot \left[\frac{F_1(m^2_{\eta_c'})}{F_1(m^2_{\eta_c})}\right]^2 \cdot \left[\frac{m_B^2 - m^2_{\eta_c'}}{m_B^2 - m^2_{\eta_c}}\right]^3 \\ &\approx 0.9 \times \left(\frac{f_{\eta_c'}}{f_{\eta_c}}\right)^2 \approx 0.45, \end{aligned} \tag{17}$$

where we have used $f_{\eta_c'}/f_{\eta_c} \approx f_{\psi'}/f_{J/\psi}$ with $f_{J/\psi} = 400\,\mathrm{MeV}$, $f_{\psi'} = 280\,\mathrm{MeV}$, which are determined from the observed leptonic decay widths [13], and $F_0(p^2) = (1 - p^2/m_B^2)F_1(p^2)$ with $F_1(m^2_{\eta_c'}) = 0.81$, $F_1(m^2_{\eta_c}) = 0.58$ [3, 11].

The ratio in (17) is valid for leading order and will roughly hold even when we include the $\mathcal{O}(\alpha_s)$ corrections, because the $\mathcal{O}(\alpha_s)$ corrections are small and the mass difference as well as the wave function difference between η_c and η_c' will not change the values of a_i in (10) greatly. Then we get the decay branching ratio as

$$\mathrm{Br}(B^0 \to \eta_c' K^0) \approx 0.7 \times 10^{-4}. \tag{18}$$

The Belle Collaboration has reported the observation of the η_c' in exclusive $B \to K K_S K^- \pi^+$ decays [9]:

$$\frac{\mathrm{Br}(B^0 \to \eta_c' K^0) \cdot \mathrm{Br}(\eta_c' \to K_S K^- \pi^+)}{\mathrm{Br}(B^0 \to \eta_c K^0) \cdot \mathrm{Br}(\eta_c \to K_S K^- \pi^+)} = 0.38 \pm 0.12 \pm 0.05. \tag{19}$$

As was noted in [14], the hadronic decay branching fractions for η_c and η_c' are expected to be roughly equal for the helicity non-suppressed decay channels[1]. So we have

$$\mathrm{Br}(\eta_c' \to K_S K^- \pi^+) \approx \mathrm{Br}(\eta_c \to K_S K^- \pi^+). \tag{20}$$

Then from (19) we get

$$\frac{\mathrm{Br}(B^0 \to \eta_c' K^0)}{\mathrm{Br}(B^0 \to \eta_c K^0)} \approx 0.4, \tag{21}$$

which is consistent with the theoretical estimate given in (17).

[1] It will also be interesting to detect the helicity suppressed decay channels of η_c and η_c' in B decays, and to see the differences between the helicity suppressed (e.g. $\rho\rho, K^*\bar{K}^*, \phi\phi, p\bar{p}$) and non-suppressed (e.g. $K\bar{K}\pi, \eta\pi\pi, \eta'\pi\pi$) decays of η_c and η_c'. This will be useful to clarify the helicity suppression mechanism for the charmonium hadronic decays and the so-called $\rho\pi$ puzzle in J/ψ and ψ' decays observed by BES and MARKII in e^+e^- annihilation experiments. For details, see [14, 15].

However, as was mentioned in Sect. 2, because the theoretical decay branching ratio is about seven times smaller than the experimental data for $B^0 \to \eta_c K^0$ decay, the theoretical branching fraction will also be about seven times smaller than the experimental data for $B^0 \to \eta_c' K^0$ decay.

4 Discussion

We have shown that for B decays to η_c and η_c' the theoretical branching fractions are all about seven times smaller than the experimental data. However, from (17) and (21) we see that the theoretical ratio of the decay rates of the two states is consistent with the experimental data:

$$\frac{\mathrm{Br}(B^0 \to \eta_c' K^0)}{\mathrm{Br}(B^0 \to \eta_c K^0)}_{\mathrm{Th.}} \approx \frac{\mathrm{Br}(B^0 \to \eta_c' K^0)}{\mathrm{Br}(B^0 \to \eta_c K^0)}_{\mathrm{Ex.}} . \tag{22}$$

It is also interesting to find that although the theoretical branching fractions of B meson exclusive decays to J/ψ and ψ' are both much smaller than the experimental data, the theoretical ratio of the decay rates of these two states is also roughly consistent with the experimental data [13]:

$$\frac{\mathrm{Br}(B^0 \to \psi' K^0)}{\mathrm{Br}(B^0 \to J/\psi K^0)}_{\mathrm{Th.}}$$

$$\approx \left(\frac{f_{\psi'}}{f_{J/\psi}}\right)^2 \cdot \left[\frac{F_1(m_{\psi'}^2)}{F_1(m_{J/\psi}^2)}\right]^2 \cdot \left[\frac{m_B^2 - m_{\psi'}^2}{m_B^2 - m_{J/\psi}^2}\right]^3$$

$$\approx 0.9 \times \left(\frac{f_{\psi'}}{f_{J/\psi}}\right)^2 \approx 0.45, \tag{23}$$

$$\frac{\mathrm{Br}(B^0 \to \psi' K^0)}{\mathrm{Br}(B^0 \to J/\psi K^0)}_{\mathrm{Ex.}} \approx 0.6, \tag{24}$$

where $F_1(m_{\psi'}^2) = 0.83$ and $F_1(m_{J/\psi}^2) = 0.61$ are used.

Another interesting observation is that the theoretical ratio of the branching fractions of B meson exclusive decays to η_c and J/ψ is also roughly consistent with the experimental data [6–8, 13]:

$$\frac{\mathrm{Br}(B^0 \to \eta_c K^0)}{\mathrm{Br}(B^0 \to J/\psi K^0)}_{\mathrm{Th.}}$$

$$\approx \left(\frac{f_{\eta_c}}{f_{J/\psi}}\right)^2 \cdot \left[\frac{F_1(m_{\eta_c}^2)}{F_1(m_{J/\psi}^2)}\right]^2 \cdot \left[\frac{m_B^2 - m_{\eta_c}^2}{m_B^2 - m_{J/\psi}^2}\right]^3$$

$$\approx 1.1 \times \left(\frac{f_{\eta_c}}{f_{J/\psi}}\right)^2 \approx 0.83\text{–}1.2, \tag{25}$$

$$\frac{\mathrm{Br}(B^0 \to \eta_c K^0)}{\mathrm{Br}(B^0 \to J/\psi K^0)}_{\mathrm{Ex.}} \approx 1.0. \tag{26}$$

The ratio in (25) ranges with the ratio of the squared decay constants taken from 0.75 to 1.1 in various models, and becomes slightly larger when $\mathcal{O}(\alpha_s)$ corrections are included.

So the predicted relative rates of all S-wave charmonium states $J/\psi, \psi', \eta_c, \eta_c'$ in the QCD factorization approach are roughly compatible with the data. This has been shown explicitly above in the leading-order approximation, and even holds when including $\mathcal{O}(\alpha_s)$ corrections with which the calculated decay rates for these four charmonium states are almost equally smaller than data by a factor of 6–9 with some theoretical uncertainties associated with form factors[2], decay constants, as well as the light-cone wave functions of meson involved. This result is rather puzzling, and it might imply that the naive factorization for B decays to the S-wave charmonia may still make sense but the overall normalizations for the decay rates are questionable.

It is worthwhile to notice that for the $c\bar{c}$ system in the non-relativistic limit we will get the same decay rates by adopting the non-relativistic bound-state picture for describing the charmonia as that by choosing the light-cone distribution amplitude as we have done above. In fact, in the non-relativistic approximation, when the relative momentum between the c and anti-c quarks inside the S-wave charmonia is neglected, the Bethe–Salpeter wave function for η_c can be reduced to its non-relativistic form and the decay amplitude will be proportional to the η_c wave function at the origin which is related to the η_c decay constant defined in (4) by

$$f_{\eta_c} = \sqrt{\frac{3}{\pi m_{\eta_c}}} R(0), \tag{27}$$

where $R(r)$ is the η_c radial wave function. Then it is easy to see that in the leading order the non-relativistic wave function of η_c corresponds to the light-cone distribution amplitude defined in (5) with the choice $\phi_{\eta_c}(u) = \delta(u - 1/2)$.

In summary, we have studied the exclusive decays of the B meson into pseudoscalar charmonium states η_c and η_c' within the QCD factorization approach and find that the non-factorizable corrections to naive factorization are infrared safe at leading-twist order. The spectator interactions arising from the kaon twist-3 effects are formally power suppressed but chirally and logarithmically enhanced. The theoretical decay rates are too small to accommodate the experimental data. An important improvement of QCD factorization compared with naive factorization is that the dependence of the branching ratio on the renormalization scale μ becomes very mild when we include the

[2] In our calculation, we have used the relation $F_0(p^2) = (1 - p^2/m_B^2)F_1(p^2)$ derived in [3], which is consistent with the form factors obtained in [11]. This will reduce the effects of uncertainties arising from form factors on the decay rate ratios. For example, in (25) we have used

$$\left(\frac{F_0(m_{\eta_c}^2)}{F_1(m_{J/\psi}^2)}\right)^2 = (1 - m_{\eta_c}^2/m_B^2)^2 \left(\frac{F_1(m_{\eta_c}^2)}{F_1(m_{J/\psi}^2)}\right)^2$$

$$\approx (1 - m_{\eta_c}^2/m_B^2)^2 = 0.46.$$

This value is close to that given in [16], which is the modified version of [17]. In [16], the authors discussed the B decay rate ratio of η_c to J/ψ at the leading order and assumed that $F_0(p^2)$ is a constant and $F_1(p^2)$ has a monopole dependence with specific pole masses.

non-factorizable $\mathcal{O}(\alpha_s)$ corrections. We already knew that for the $B \to J/\psi K$ decay, there are also logarithmic divergences arising from spectator interactions due to kaon twist-3 effects and the calculated rates are also smaller than the data by a factor of 8–10 [3, 4]. However, it might be interesting to note that the predicted relative rates of all S-wave charmonium states $J/\psi, \psi', \eta_c, \eta_c'$ within the QCD factorization approach are roughly compatible with the experimental data. In conclusion, the QCD factorization approach is applicable for B exclusive decays to the S-wave charmonia, but further study on the higher-twist contributions from charmonium light-cone distribution amplitudes as well as other new ingredients or mechanisms in exclusive non-leptonic B meson decays to charmonium states are needed to clarify the discrepancy between theoretical calculations and experimental data.

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